COMPARISONS OF ESTIMATES IN TWO-STAGE SAMPLING ON SUCCESSIVE OCCASIONS

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1. INTRODUCTION AND SUMMARY

The theory of successive sampling developed by Jessen (1942), Yates (1960), Patterson (1950), Eckler (1955), Rao and Graham (1964) and others for single-stage sampling design was extended to two-stage sampling by Kathuria (1959), Singh (1968) and Singh and Kathuria (1969). Tikkiwal (1964) has extended the covariance conditions developed by Patterson to two-stage sampling and obtained conditions for arriving at best unbiased estimator for the mean on the current occasion.

Using a two-stage sampling design, Singh and Kathuria (1969) obtained the minimum-variance unbiased estimator of mean under two different sampling patterns, namely, (I) when a fraction p of the primary sampling units (psu's) with their samples of second stage units (ssu's) are retained from previous occasion to the current occasions and a fraction q of the psu's are selected afresh ($q+p=1$), and (II) when all the psu's in the sample on the previous occasion are retained on current occasion but only a fraction p of the sample of ssu's within each psu is retained and a fraction q of the ssu's is selected afresh.

In repeat surveys, one is often interested not only in obtaining an estimate of the character for the most recent occasion but also in study of change in value of the character on the recent occasion from the immediately preceding occasion. Similarly, it would be of interest to simultaneously estimate the mean on the current occasion and the change on the current occasion from the immediately preceding occasion.

In section 2.1 we define the population parameters and indicate the assumptions made on them. In sections 2.2 and 2 3 are given the sample estimates of the population means and their variances derived under the two sampling patterns as obtained by Singh and Kathuria (1969) and in section 2.4 the relative efficiency of the two estimates has been obtained in sampling upto 5 occasions. In sections 3 and 4, estimate of change and simultaneous estimation of mean and change have been discussed under the two sampling patterns and their relative efficiencies have been obtained. It would be seen that for estimating the mean on the current occasion, sampling pettern I is generally better than the sampling pattern II so long as the mean squares between psu means is larger than the mean squares between ssu's and the correlation between the psu means is not equal to zero. Similarly, for estimating the change as well as for obtaining the combined estimate of the mean and the change sampling pattern II is found better than pattern I for some values of the correlation coefficients and the mean squares between psu's and ssu's and *vice-versa* for others.

2. Estimate of mean on the current occasion

2.1. Let there be a population π consisting of N psu's wherein each psu consists of M ssu's. Let $y_{kl}^{(i)}$ be the value of the l-th ssu in the k-th psu drawn on the i-th occasion ($i=1, 2,\ldots h$; $k=1, 2,\ldots N$; $l=1, 2,\ldots M$). We are required to estimate the population mean $\overline{Y}_{..}^{(h)}$, where

$$\overline{Y}_{..}^{(h)}=\frac{1}{NM}\sum_{k=1}^{N}\sum_{l=1}^{M}y_{kl}^{(h)}.$$

Define

$$S_b^{2(i)}=\frac{1}{(N-1)}\sum_{k=1}^{N}\left(\overline{Y}_{k.}^{(i)}-\overline{Y}_{..}^{(i)}\right)^2$$

as the mean square between the psu means in the population on the i-th occasion,

$$S_w^{2(i)}=\frac{1}{N(M-1)}\sum_{k=1}^{N}\sum_{l=1}^{M}\left(y_{kl}^{(i)}-\overline{Y}_{k.}^{(i)}\right)^2$$

as the mean square between ssu's within psu's in the population on the i-th occasion ($i=1, 2, \ldots h$),

$$P_b^{(i,j)}S_b^{(i)}S_b^{(j)}=\frac{1}{(N-1)}\sum_{k=1}^{N}\left(\overline{Y}_{k.}^{(i)}-\overline{Y}_{..}^{(i)}\right)\left(\overline{Y}_{k.}^{(j)}-\overline{Y}_{..}^{(j)}\right)$$

as the covariance between psu means in the population on the i-th and j-th occasions and

$$\rho_w^{(i,j)} S_w^{(i)} \cdot S_w^{(j)} = \frac{1}{N(M-1)} \sum_{k=1}^{N} \sum_{l=1}^{M} \left(y_{kl}^{(i)} - \overline{Y}_{k.}^{(i)}\right)\left(y_{kl}^{(j)} - \overline{Y}_{k.}^{(j)}\right)$$

as the covariance between ssu's within psu's on the i-th and j-th occasions ($i \neq j = 1, 2, \ldots h$). For simplicity, we may assume N and M to be large such that terms of the order $\frac{1}{N}$ and $\frac{1}{M}$ are ignored.

Further, we may assume that

$$S_b^{(i)} = S_b^{(j)} = S_b \text{ and } S_w^{(i)} = S_w^{(j)} = S_w.$$

We also assume that

$$\rho_b^{(i,j)} = \rho_b^{|i j|} \text{ and } \rho_w^{(ij)} = \rho_w^{|i j|}$$

for all $i \neq j = 1, 2, \ldots h.$

Also we write

$$S_b^2 + \frac{S_w^2}{m} = \alpha, \; \rho_b S_b^2 + \rho_w \frac{S_w^2}{m} = \gamma.$$

2.2. Sampling pattern I

Let the sample size on any occasion consist of n psu's each of them consisting of m ssu's. Let the sample size on the h-th occasion consist of np psu's with their samples of ssu's retained from the total sample drawn on the preceding occasion and nq psu's selected afresh from the population ($q + p = 1$).

Let $\overline{y}_1^{(h-1)} \overline{y}_1^{(h)}$ and $\overline{y}_2^{(h-1)}, \overline{y}_2^{(h)}$ denote the means on the ($h-1$)-th and h-th occasions based on npm and nqm units respectively. Singh and Kathuria (1969) obtained the following estimate of $\overline{Y}^{(h)}$.

$$\overline{y}^{(h)} = c_h \left[\overline{y}_1^{(h)} + \frac{\gamma}{\alpha}\left(\overline{y}^{(h-1)} - \overline{y}_1^{(h-1)} \right)\right] + (1 - c_h)\, \overline{y}_2^{(h)} \quad \ldots(2.2.1)$$

where

$$c_h = \frac{p}{1 - \frac{\gamma^2}{\alpha^2}(q-p) - pc_{h-1}\frac{\gamma^2}{\alpha^2}} \qquad \text{...(2.2.2.)}$$

and $c_1 = p$. The variance of the estimate $\overline{y}^{(h)}$ is given by

$$V(\overline{y}^{(h)}) = (1-c_h)\frac{\alpha}{nq}. \qquad \text{...(2.2.3)}$$

The limiting value of c_h when sampling is carried over sufficient number of occasions may be obtained by writing $c_h = c_{h-1} = c$ in (2.2.2) above and then solving for c.

This gives

$$c = \frac{[\alpha^2 - \gamma^2(q-p)] - \sqrt{[\alpha^2 - \gamma^2(q-p)]^2 - 4p^2\alpha^2\gamma^2}}{2pr^2} \qquad \text{...(2.2.4)}$$

as the possible value of

2.3. SAMPLING PATTERN II

With the same notation for the sample means as in section 2.2., the estimate of the population mean on the h-th occasion may be obtained as follow :

$$E^{(h)} = b_h\left\{\overline{y}_1^{(h)} + \rho_w\left(\overline{y}^{(h-1)} - \overline{y}_1^{(h-1)}\right)\right\} + (1-b_h)\,\overline{y}_2^{(h)} \qquad \text{...(2·3·1)}$$

and its variance is given by

$$V(E^{(h)}) = \frac{S_b^2}{n} + (1-b_h)\frac{S_w^2}{nmq} \qquad \text{...(2·3·2)}$$

where

$$b_h = \frac{p}{1-(q-p)\rho_w^2 - pb_{h-1}\rho_w^2}; \qquad \text{...(2·3·3)}$$

b_1 being equal to p.

The limiting value of b_h when sampling is carried over sufficient number of occasions is obtained by writing $b_h = b_{h-1} = b_1$ in (2 3·3) and then solving for b which is given by

$$b = \frac{[1-\rho_w^2(q-p)] - \sqrt{[1-\rho_w^2(q-p)]^2 - 4p^2\rho_w^2}}{2p\rho_w^2} \qquad \text{(2·3·4)}$$

2·4. *Efficiencies of the estimates and optimum replacement fraction :*

Singh and Kathuria (1969) examined the efficiency of the estimate $E^{(2)}$ in relation to $\overline{y}^{(2)}$ *i.e.* for $h=2$, for a set of values of

$$\rho_b, \rho_w, \phi=\left(\frac{S_w^2}{S_b^2}\right)$$

and m. Only positive values of ρ_b and ρ_w were taken although negative values of ρ_b and ρ_w in an actual survey are not un-common.

In table 2·1, the efficiency of the estimate $E^{(h)}$ has been worked out in relation to $\overline{y}^{(h)}$ for $h=2$, 3, 4 and 5 for a set of values of ρ_b. ρ_w and ϕ. It would be seen from (2·3·2) and by virtue of (2 3·3) that the relative efficiencies are symmetrical w.r.t. ρ_w. Hence only positive values of ρ_w, ranging from 0 to 0·9 have been taken in Table 2·1.

The following observations may be made regarding the efficiency of the estimate $E^{(h)}$ in relation to the estimate $\overline{y}^{(h)}$.

(*i*) When $\phi=0{\cdot}1$ and hence $S_w^2 < S_b^2$, $E^{(h)}$ is less efficient than $\overline{y}^{(h)}$ for all h and for all values of ρ_b and ρ_w except when $\rho_b=0$ in which case both the estimates are equally efficient. There is a slight decrease in the relative efficiency when we go beyond the second occasion.

(*ii*) When $\phi=1$, *i.e.*, $S_w^2=S_b^2$, $E^{(h)}$ is more efficient than $\overline{y}^{(h)}$ only for particular combinations of $\rho_b \leqslant 0$ and $\rho_w \geqslant 0{\cdot}7$ for all h. The efficiency slightly increases or remains constant from third occasion and onwards.

(*iii*) When $\phi=10$ and hence

$$S_w^2 < S_b^2$$

$E^{(h)}$ is more efficient than $\overline{y}^{(h)}$ when $\rho_b \leqslant 0$ and $\rho_w \geqslant 0$. $E^{(h)}$ is also equal to or a little more efficient than $\overline{y}^{(h)}$ when $\rho_b=0{\cdot}5$ and $\rho_w=0$ and 0·9.

When $h=2$, using sampling pattern I we have

$$V(\overline{y}^{(2)}) = \frac{\alpha}{n} \cdot \frac{(\alpha^2-\gamma^2 q)}{(\alpha^2-\gamma^2 q^2)} \qquad \ldots(2{\cdot}4{\cdot}1)$$

The optimum value of q [call it $q(1)$] is given by

$$\frac{d}{dq}(V\bar{y}^{(2)})=0.$$

This gives

$$q_{(1)}=\frac{\alpha^2-\alpha\sqrt{\alpha^2-\gamma^2}}{\gamma^2} \qquad ...(2\cdot4\cdot2)$$

and

$$V_{\text{opt}}(\bar{y}_b^{(2)})=\frac{\alpha}{2n}\left[1+\frac{\sqrt{\gamma^2}}{\alpha^2}\right] \qquad ...(2\cdot4\cdot3)$$

Similarly, using sampling pattern II we have

$$V(E^{(2)})=\frac{S_b^2}{n}+\frac{(1-\rho_w^2q)}{(1-\rho_w^2q^2)}\frac{S_w^2}{nm} \qquad ...(2\cdot4\cdot4)$$

The optimum replacement fraction, call it $q_{(2)}$, may be seen to be given by

$$q_{(2)}=\frac{1-\sqrt{1-\rho_w^2}}{\rho_w^2} \qquad ...(2\cdot4\cdot5)$$

and

$$V_{\text{opt.}}(E^{(2)})=\frac{S_b^2}{n}+\left(1+\sqrt{1-\rho_w^2}\right)\frac{S_w^2}{2nm}. \qquad ...(2\cdot4\cdot6)$$

3. Estimates of change :

3·1. *Sampling pattern I*

We estimate the difference $\overline{Y}^{(h)}-\overline{Y}^{(h-1)}$ of the means of populations on hth and $(h-1)$-th occasion by the quantity.

$$d^{(h)}=\bar{y}^{(h)}-\bar{y}^{(h-1)}. \qquad ...(3\cdot1\cdot1)$$

To obtain $V(d^{(h)})$ we need to get

$$\text{Cov}\,(\bar{y}^{(h)},\ \bar{y}^{(h-1)}).$$

Now,

$$\text{Cov}\,(\overline{Y}^{(h)}\ \bar{y}^{(h-1)})=\text{Cov}\left[\bar{y}^{(h-1)},\ c_h\left\{\bar{y}_1^{(h)}+\frac{\gamma}{\alpha}(\bar{y}^{(h-1)}-\bar{y}_1^{(h-1)})\right\}+(1-c_h)\bar{y}_2^{(h)}\right]=c_h(1-c_{h-1}).\frac{\gamma}{nq}$$

Therefore,

$$V(d^{(h)}) = V(\bar{y}^{(h)}) + V(\bar{y}^{(h-1)}) - 2\text{Cov}\,(\bar{y}^{(h)},\ \bar{y}^{(h-1)})$$

$$= (1-c_h)\frac{\alpha}{np} + (1-c_{h-1})\frac{\alpha}{np} - 2c_h(1-c_h-1)\frac{\Upsilon}{nq}$$

$$= 2(1-c)\left(1-\frac{\Upsilon}{\alpha}c\right)\frac{\alpha}{nq}\ ; \qquad \ldots(3\cdot1\cdot2)$$

where c is given by (2·2 4) above.

3·2. *Sampling pattern II*

We write the estimate of difference of means $\overline{Y}^{(h)}$ and $\overline{Y}^{(h-1)}$ as

$$D^{(h)} = E^{(h)} - E^{(h-1)}. \qquad \ldots(3\cdot21)$$

Now,

$$\text{Cov}\ (E^{(h)},\ E^{(h-1)}) = \rho_b\frac{S_b^2}{n} + b_h(1-b_{h-1})\,\rho_w\frac{S_w^2}{nmq}.$$

Therefore,

$$V(D^{(h)}) = V(E^{(h)}) + V(E^{(h-1)}) - 2\ \text{Cov}\ (E^{(h)},\ E^{(h-1)}),$$

$$= 2(1-\rho_b)\frac{S_b^2}{n} + 2(1-b)(1-b\,\rho_w)\frac{S_w^2}{nmq} \qquad \ldots(3\cdot2\cdot2)$$

where b is given by (2·3·4) above. In table 3·1, the efficiency of the estimate $D^{(h)}$ has been worked out in relation to the estimate $d^{(h)}$ for a set of values of ρ_b, ρ_w, ϕ and m.

The following observations may be made regarding the efficiency of the estimate $D^{(h)}$ in relation to the estimate $d^{(h)}$.

(*i*) When $\rho_b \geqslant 0$, $D^{(h)}$ is more efficient than $d^{(h)}$. $D^{(h)}$ is also more efficient than $d^{(h)}$ for $\rho_b < 0$ when $\rho_w = 0{\cdot}9$ and $S_w^2 > S_b^2$.

(*ii*) For a given value of $\phi\left(=\frac{S_w^2}{S_b^2}\right)$, the efficiency increases as ρ_b increases from $-0{\cdot}9$ to $+0{\cdot}9$.

(*iii*) As ϕ increases from 0·1 to 10·0, the relative efficiency increases if $\rho_b \leqslant 0$ but decreases if $\rho_b > 0$ for fixed values of ρ_w and q.

(*iv*) The relative efficiency decreases as q changes from 0·5 to 0·75 when $\rho_b < 0$ but increases when $\rho_b \geqslant 0$.

4. Combined estimate of mean and difference

4·1. Let us now consider the case when estimate of mean on the current occasion and the difference of means between two occasions are both of equal interest. For simplicity sake, we will take the case when sampling is done for two occasions. The objective is to obtain an estimate $\bar{z}^{(2)}$ for mean on the second occasion and an estimate $\bar{d}^{(2)}$ of change such that the estimated change is the difference between estimated means *i.e.* $\bar{d}^{(2)}=\bar{z}^{(2)}-\bar{y}^{(1)}$, where $\bar{y}^{(1)}$ is the sample mean on the first occasion. It may be easily seen that $\bar{z}^{(2)}$ and $\bar{d}^{(2)}$ can be written in the form :

$$\bar{z}^{(2)}=u_1\bar{y}_1{}^{(2)}+(1-u_1)\bar{y}_2{}^{(2)}+(v_1+p)(\bar{y}_1{}^{(1)}-\bar{y}_2{}^{(1)}) \quad \text{...(4·1·1)}$$

and

$$\bar{d}^{(2)}=u_1\bar{y}_1{}^{(2)}+(1-u_1)\bar{y}_2{}^{(2)}+v_1\bar{y}_1{}^{(1)}-(1+v_1)\bar{y}_2{}^{(1)} \quad \text{...(4·1·2)}$$

where u_1 and v_1 may be obtained in a manner such that $V(\bar{z}^{(2)})+$ $+V(\bar{d}^{(2)})$ is minimum. Under sampling pattern I, the values of u_1 and v_1 are given by

$$u_1=\frac{p(\alpha^2+\frac{1}{2}q\gamma\alpha)}{(\alpha^2-\gamma^2q^2)}$$

and

$$v_1=\frac{p(\frac{1}{2}\gamma^2q^2-\alpha\gamma q-\alpha^2)}{(\alpha^2-\gamma^2q^2)}$$

With the above values of u_2 and v_1, we get

$$V(\bar{z}^{(2)})=\frac{\alpha}{n}\,\frac{(\alpha^2-\gamma^2q)}{(\alpha^2-\gamma^2q^2)}+\frac{1}{4n}\,\frac{pq\alpha\gamma^2}{(\alpha^2-\gamma^2q^2)} \quad \text{...(4·1·3)}$$

and

$$V(\bar{d}^{(2)})=\frac{2\alpha(\alpha-\gamma)}{n(\alpha-\gamma q)}+\frac{1}{4n}\,\frac{pq\alpha\gamma^2}{(\alpha^2-\gamma^2q^2)} \quad \text{...(4·1·4)}$$

The optimum value of q which minimises $V(\bar{z}^{(2)})$ is given by

$$q_{(1)}=\frac{\alpha^2-\alpha\sqrt[4]{\alpha^2-\gamma^2}}{\gamma^2}$$

and the optimum variance is given by

$$V_{opt.}(\bar{z}^{(2)})=\frac{1}{2n}\,(\alpha+\sqrt{\alpha^2-\gamma^2})+\frac{\gamma}{8n(\alpha+\sqrt{\alpha^2-\gamma^2})} \quad \text{...(4·1·3)}'$$

Similarly, the optimum value of q which would minimise $V(\bar{d}^{(2)})$ is given by $q'_{(1)}=0$ and the optimum variance is

$$V_{opt.}(\bar{d}^{(2)})=\frac{2}{n}(\alpha-\gamma). \qquad \ldots(4\cdot1\cdot4)'$$

Under sampling pattern II, the estimates of mean and the difference may be written as

$$\bar{Z}^{(2)}=u_2\bar{y}_1^{(2)}+(1-u_2)\bar{y}_2^{(2)}+(v_2+p)(\bar{y}_1^{(1)}-\bar{y}_2^{(1)}) \qquad \ldots(4\cdot1\cdot5)$$

and

$$\bar{D}^{(2)}=u_2\bar{y}_1^{(2)}+(1-u_2)\,\bar{y}_2^{(2)}+v_2\bar{y}_1^{(1)}-(1+v_2)\bar{y}_2^{(2)} \qquad \ldots(4\cdot1\cdot6)$$

where u_2 and v_2 are obtained in the same manner in which u_1 and v_1 were obtained. This gives.

$$u_2=\frac{p(1+\frac{1}{2}\rho_w q)}{(1-\rho_w^2 q^2)}$$

and

$$v_2=\frac{p(\frac{1}{2}\rho_w^2 q^2-\rho_w q-1)}{(1-\rho_w^2 q^2)}$$

The minimum variances of the estimates are given by

$$V(\bar{z}^{(2)})=\frac{S_b^2}{n}+\frac{(1-\rho_w^2 q)}{(1-\rho_w^2 q^2)}\frac{S_w^2}{nm}+\frac{pq\rho_w^2 S_w^2}{4(1-\rho_w^2 q^2)nm} \qquad \ldots(4\cdot1\cdot7)$$

and

$$V(\bar{D}^{(2)})=\frac{2}{n}(1-\rho_b)S_b^2+\frac{3(1-\rho_w)}{(1-\rho_w q)}\frac{S_w^2}{nm}+\frac{pq\rho_w^2}{4(1-\rho_w^2 q^2)}\frac{S_w^2}{nm} \qquad \ldots(4\cdot1\cdot8)$$

The optimum value of q which minimises $V(\bar{Z}^{(2)})$ is given by

$$q_{(2)}=\frac{1-\sqrt{1-\rho_w^2}}{\rho_w^2}$$

and the optimum variance is given by

$$V_{opt\cdot\rho}(\bar{Z}^{(2)})=\frac{S_b^2}{n}+(1+\sqrt{1-\rho_w^2})\frac{S_w^2}{2nm}+\frac{\rho_w^2 S_w^2}{8nm(1+\sqrt{1-\rho_w^2})} \qquad \ldots(4\cdot1\cdot7)'$$

Similarly, the optimum value of q which minimises $V(\bar{D}^{(2)})$ is given by $q'_{(2)}=0$ and the optimum variance is given by

$$V_{opt.}(\bar{D}^{(2)})=\frac{2}{n}(1-\rho_b)\,S_b^2+2(1-\rho_w)\frac{S_w^2}{nm} \qquad \ldots(4\cdot1\cdot8)'$$

which may be seen to be same as (4·1·4)′ above. By comparing (4·1·3) with (2·4·1) and (4·1·7) with (2·4·4) it may be seen that if the objective of the survey is to obtain combined estimate of both the mean and the difference, the variances of the estimated mean is increased by the quantity

$$\frac{pq\alpha\gamma^2}{4nm(\alpha^2-\gamma^2q^2)}$$

when sampling pattern I is used and is increased by the quantity

$$\frac{pq\rho_w^2S_w^2}{4(1-\rho_w^2q^2)nm}$$

when sampling pattern II is used. Taking into account the optimum values of q, under sampling pattern I, (4·1·3)′ is larger than (2·4·3) by the quantity

$$\frac{\gamma^2}{8n(\alpha+\sqrt{\alpha^2-\gamma^2})}$$

and under sampling pattern II, (4·1·7)′ is larger than (2·4·6) by the quantity

$$\frac{\rho_w^2\, S_w^2}{8nm(1+\sqrt{1-\rho_w^2})}.$$

Kathuria (1959) obtained estimates of difference of means of two occasions under both the sampling patterns with the assumptions made in section 2·1. The variances of these estimates are given by

$$V_I = \frac{2\alpha(\alpha-\gamma)}{n(\alpha-\gamma q)}$$

and

$$V_{II} = \frac{2(1-\rho_b)S_b^2+2(1-\rho_w)}{n(1-\rho_w q)} \frac{S_w^2}{nm}$$

where suffices I and II in the above variance expressions indicate the corresponding sampling patterns. It may be seen that the increase in variance of estimates of difference under both the sampling patterns is the same as for respective estimates of mean. The relative efficiencies of the estimates of mean and the difference under the two sampling patterns may be seen in Tables 4·1, and Table 4·2.

We make the following obsevations regarding the efficiency of the sampling pattern I with respect to sampling patterns II regarding the combined estimates of mean and change.

(*i*) In order to estimate the mean, for $\phi < 1$ (*i.e.* $S_w^2 < S_b^2$), sampling pattern II is generally better for all values of ρ_b and ρ_w except for $\rho_b=0$. For $\rho_b=0$, the two patterns are equally efficient.

(*ii*) For $\phi \geqslant 1$ (*i.e.* $S_w^2 \geqslant S_b^2$), sampling pattern II is better than pattern I for $\rho_b \geqslant 0$, $\rho_w \leqslant 0$ and $\rho_b \leqslant 0$, $\rho_w \geqslant 0$. and *vice-versa* for $\rho_b \leqslant 0$, $\rho_w \leqslant 0$ and $\rho_b \geqslant 0$, $\rho_w \geqslant 0$.

(*iii*) Similarly for estimating the change, sampling pattern II is better than pattern I for $\rho_b < 0$ for all values of ρ_w and ϕ. For $\rho_b \geqslant 0$, sampling pattern I is better that pattern II.

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TABLE 2.1

Relative efficiency of the estimate $E^{(h)}$ w.r.t. the estimate $\overline{y}^{(h)}(h=2, 3, 4, 5)$ for different values of ρ_b, ρ_w, ϕ and for $m=4$

h	$\rho_w=$ ρ_b	$q=0.50$																			
		$\phi=0\ 1$				$\phi=0.25$				$\phi=1$				$\phi=4$				$\phi=10$			
		0.0	0.5	0.7	0.9	0.00	0.5	0.7	9.9	0.0	0.5	0.7	0.9	0.0	0.5	0.7	0.9	0.0	0.5	0.7	0.9
		1				2				3				4				5			
	−0.9	0.76	0.77	0.78	0.78	0.78	0.80	0.81	0.82	0.85	0.91	0.93	0.97	0.95	1.02	1.07	1.14	0.98	1.05	1.09	1.17
	−0.7	0.87	0.87	0.88	0.88	0.88	0.89	0.90	0.91	0.91	0.96	0.98	1.01	0.97	1.03	1.07	1.14	0.99	1.04	1.08	1.16
	−0.5	0.94	0.94	0.94	0.95	0.94	0.95	0.96	0.97	0.96	0.99	1.01	1.04	0.98	1.03	1.07	1.13	0.99	1.04	1.07	1.14
2	0.0	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.01	1.00	1.01	1.02	1.04	1.00	1.02	1.04	1.08	1.00	1.02	1.04	1.08
	0.5	0.94	0.93	0.94	0.94	0.94	0.94	0.94	0.94	0.96	0.94	0.95	0.96	0.98	0.97	0.97	0.98	0.99	0.98	0.98	1.00
	0.7	0.87	0.86	0.86	0.86	0.88	0.87	0.87	0.87	0.91	0.89	0.88	0.89	0.97	0.93	0.92	0.93	0.99	0.96	0.96	0.96
	0.9	0.76	0.75	0.75	0.75	0.78	0.76	0.76	0.76	0.85	0.81	0.80	0.78	0.95	0.89	0.87	0.85	0.98	0.94	0.92	0.91
	−0.9	0.69	0.70	0.71	0.72	0.72	0.75	0.77	0.88	0.82	0.89	0.93	0.98	0.94	1.03	1.09	1.20	0.98	1.05	1.11	1.27
	−0.7	0.84	0.86	0.86	0.87	0.86	0.88	0.89	0.91	0.91	0.95	0.98	1.03	0.97	1.03	1.09	1.20	0.99	1.05	1.10	1.25
	−0.5	0.93	0.94	9.94	0.95	0.94	0.95	0.96	0.97	0.96	0.99	1.01	1.06	0.98	1.04	1.09	1.19	0.99	1.04	1.09	1.23
3	0.0	0.00	1.00	1.00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.03	1.06	1.00	1.02	1.05	1.14	1.00	1.02	1.05	1.15
	0.5	1.93	0.93	0.93	0.93	0.94	0.93	0.93	0.94	0.96	0.94	0.94	0.96	0.98	0.96	0.97	1.01	0.99	0.98	0.98	1.03
	0.7	0.85	0,84	0.84	0.84	0.86	0.85	0.85	0,35	0.91	0.87	0.86	0.87	0.97	0.92	0.91	0.92	0.99	0.96	0.95	0.95
	0.9	0.69	0.67	0.67	0.66	0,72	0.69	0.68	0.67	0.82	0.76	0.73	0.71	0.94	0.87	0.83	0.79	0.98	0.93	0.90	0.87

		1				2				3				4				5			
	−0.9	0.66	0.68	1.69	0.70	0.70	0.74	0.76	0.78	0.82	0.89	0.93	0.99	0.94	1.03	1.09	1.23	0.98	1.05	1.12	1.31
	−0.7	0.84	0.85	0.86	0.86	0.86	0.88	0 89	0.91	0.91	0.95	0.98	1.04	0.97	1.03	1.09	1.23	0.99	1.05	1.11	1.29
	−0.5	0.93	0.94	0.94	0.95	0.94	0.95	0.96	0.97	0.96	0.99	1.02	1.07	0.98	1.64	1.09	1 22	0.99	1.04	1.10	1.26
4	0.0	1.00	1.00	1,00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.03	1 07	1.00	1.02	1.05	1.16	1.00	1.02	1.05	1.18
	0.5	0.93	0.93	0.93	0.93	0.94	0.93	0.93	0.94	0.96	0.94	0 94	0.97	0.98	0.95	0 97	1.02	0.99	0.98	0.98	1.04
	0.7	0.84	0.84	0.84	0.84	0.86	0.84	0.84	0.84	0.91	0.87	0.86	0.87	0.97	0.92	0.91	0.93	0.99	9.96	0.95	0.96
	0.9	0.66	0.64	0.64	0.63	0.70	0.67	0.65	0.64	0.82	0.74	0,71	0.68	0.94	0.86	0.82	0.77	0.98	0.93	0.09	0.85
	−0.9	0.65	0.67	0.68	0.69	0.73	0.73	0 75	1.77	0.82	0.89	0.93	0.99	0.94	1.03	1.09	1.24	0.98	1.05	1.12	1.32
	−0.7	0,84	0 85	0.86	0 86	0.88	0.88	0.89	0.91	0.91	0.95	0.98	1.04	0.97	1.03	1.09	1.23	0.99	1 05	1.11	1 30
	−0.5	0.93	0.94	0.94	0.95	0.95	9.95	0.96	0.97	0.96	0.99	1.02	1.07	0.98	1.04	1.09	1.23	0.99	1·04	1.10	1.28
5	0.0	1.00	1.00	1.00	1.01	1.00	1.00	1.01	0.02	1.00	1.01	1.03	1.07	1.00	1,02	1 06	1.17	1.00	1.02	1.05	1.20
	0,5	0.93	0.93	0.93	0.93	0.94	0.93	0.93	0.94	0.96	0.94	0.94	0.97	0.98	0.96	0 97	1.03	0.99	0.98	0.98	1.06
	0.7	0,84	0.84	0.84	0.84	0.86	0.84	0.84	0.84	0.91	0.87	0.86	0.87	0.97	0.92	0.91	0.93	0.99	0.96	0.94	0.97
	0 9	0.65	0.63	0.63	0.62	0.70	0.66	0.64	0.63	0.82	0.74	0.70	0.67	0.94	0.86	0,82	0.76	0.98	0.93	0.90	0.85

TABLE 2·1 (*contd.*)

h	$\rho_w=\rho_l$	q=0.75																			
		ϕ=0.1				ϕ=0.25				ϕ=1				ϕ=4				ϕ=10			
		0.0	0.5	0.7	0.9	0.0	0.5	0.7	0.9	0.0	0·5	9.7	0.9	0,0	0.5	0.7	0,9	0.0	0.5	0.7	0.9
		6				7				8				9				10			
	—0.9	0.70	0.76	0.76	0.77	0.77	0.80	0.81	0.83	0.86	0.92	0.05	0.99	0.96	1.02	1.07	1.16	0.99	1.04	1.09	1.21
	—0.7	0.87	0.89	0.89	0.90	0.89	0.91	0.92	0.93	0.93	0.96	0.99	1.03	0.97	1.03	1.07	1.16	0.99	1.04	1.08	1.20
	—0.5	0.95	0.95	0.96	0.96	0.95	0.96	0.97	0.98	0.97	0.99	1.01	1.05	0.99	1.03	1.07	1.15	1.00	1.03	1.07	1.18
2	0.0	1.00	0.00	1.00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.02	1.05	1.08	1.02	1.04	1.11	1.00	1.01	1.04	1.12
	0.5	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.97	0.96	0.96	0.98	0.99	0.98	0.97	1.01	1.00	0.98	0.99	1.03
	0.7	0.87	0.88	0.88	0.88	0.89	0.88	0.88	0.88	0.93	0.90	0.90	0.90	0.97	0.94	0.93	1.94	0.99	0.97	0.96	0.97
	0.9	0.70	0.73	0.73	0.72	0.77	0.75	0.74	0.73	0.86	0.81	0.78	0.76	0.96	0.90	0.87	0.84	0.99	0.95	0.93	0.90
	—0.2	0.70	0.72	0.73	0.74	0.74	0.78	0.79	0.81	0.85	0.91	0.94	1.00	0.96	1.02	1.07	1.20	0.99	1.04	1.09	1.27
	—0.7	0,18	0.88	0.89	0.89	0,89	0.90	0.90	9.93	0.93	0.96	1.00	1.04	0.97	1.03	1.07	1.20	0.99	1.04	1.09	1.25
	—0.5	0.95	0.95	0.95	0.96	0.95	0.96	0.97	0.98	0.97	0.99	0.01	1.06	0.99	1.03	1.07	1.19	1.00	1.03	1.08	1.24
3	0.0	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.02	1.06	1.00	1.02	1.04	1.15	1.00	1.01	1.05	1.15
	0.5	0.95	0.94	0.95	0.95	0.95	0.95	0.95	0.96	0.97	0.95	0.96	0.98	0.99	0.97	0.98	1.03	1.00	0.98	0.99	1 06
	0.7	0.87	0.87	0.87	0.87	0.89	0.87	0.87	0.88	0.93	0.90	0.89	0.90	0.97	0.94	0.93	0.95	0.99	0.96	0.96	0.98
	0.9	0.70	0.69	0.68	0,67	0.74	0.71	0.69	0.68	0.85	0.78	0.75	0.72	0.96	0.89	0.85	0.80	0.99	0.94	0.92	0.88

		6				7				8				9				10			
	—0.9	0.70	0.72	0.72	0.73	0.74	0.78	0.79	0.81	0.85	0.91	0.94	1.00	0.96	1.02	1.07	1.20	0.99	1.04	1.09	1.28
	—0.7	0.88	0.88	0.89	0.89	0.89	0.90	0.91	0.93	0.93	0.96	0.99	1.04	0.97	1.03	1.07	1.20	0.99	1.04	1.09	1.27
	—0.5	0.95	0.95	0.95	0.96	0.95	0.96	0.97	0.98	0.97	0.99	1.01	1.06	0.99	1.03	1.07	1.20	1.00	1.03	1.08	1.25
4	0.0	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.02	1.07	1.00	1.01	1.05	1.15	1.00	1.01	1.04	1.18
	0.5	0.95	0.94	0.95	0.95	0.95	0.95	0.95	0.96	0.97	0.95	0.96	0.99	0.99	0.97	0.98	1.04	1.00	0.98	0.99	1.06
	0.7	0·87	0.87	0.87	0.87	0.89	0.87	0.87	0.88	0·93	0.90	0.89	0.90	0.97	0.94	0.93	0.95	0.99	0.96	0.96	0.98
	0.9	0.70	0.58	0.67	0.66	0.74	0.70	0.68	0.67	0.85	0.78	0.74	0.71	0.96	0.89	0.85	0.79	0.99	0.94	0.92	0.97
	—0.9	0.70	0.71	0.72	0.73	0.74	0.78	0.79	0.81	0.85	0.91	0.94	1.00	0.96	1.02	1.07	1.21	0.99	1.04	1.09	1.28
	—0.7	0.87	0.88	0.89	0.99	0.87	0.90	0.91	0.93	0.93	0.96	0.99	1.04	0.97	1.03	1.07	1.20	0.99	1.04	1.09	1.27
	—0.5	0.95	0.95	0.95	0.86	0.95	0.96	0.97	0.98	0.97	0.99	1.01	1.06	0.99	1.00	1.07	1.20	1.00	1.03	1.08	1.25
5	0.0	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.02	1.00	1.01	1.02	1.07	1.00	1.02	1.04	1.15	1.00	1.01	1.04	1.19
	0.5	0.95	0.94	0.95	0.95	0.95	0.95	0.95	0.96	0.97	0.95	0.96	0.99	0.99	0.97	0.98	1.04	1.00	0.98	0.99	1.06
	0.7	0·87	0.87	0.87	0.87	0.89	0.87	0.87	0.83	0.93	0.90	0.89	0.90	0.97	0.94	0.93	0.96	0.99	0.96	0.96	0.99
	0.9	0.70	0.68	0.67	0.66	0.74	0.70	0.68	0.67	0.85	0.78	0.74	0.70	0.96	0.89	0.85	0.79	0.99	0.94	0.92	0.87

TABLE 3.1

Relative efficiency of the estimate $D^{(h)}$ of change with respect to the estimate $d^{(h)}$ for different values of P_b, P_w, ϕ and q for $m=4$

ϕ	P_b	$q=0.50$							$q=0.75$						
		$P_w=-0.9$	-0.7	-0.5	0.0	0.5	0.7	0.9	$P_w=-0.9$	-0.7	-0.5	0.0	0.5	0.7	0.9
	-0.9	0.52	0.53	0.54	0.55	0.56	0.57	0.57	0.51	0.51	0.52	0.52	0.53	0.54	0.54
	-0.7	0.70	0.70	0.70	0.70	0.70	0.71	0.71	0.64	0.64	0,64	0.64	0.64	0.65	0 65
	-0.5	0,79	0.79	0.79	0.79	0.79	0.79	0.80	0.73	0.73	0.73	9.73	0.73	0.73	0 73
0.1	0.0	1.01	1,00	1,00	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.01
	0.5	1,38	1.36	1.35	1.34	1.35	1.35	1.36	1.62	1.60	1.60	1.59	1.66	1.60	1.61
	0.7	1.68	1.64	1.63	1,61	1,61	1.62	1.64	2.18	2.15	2.13	2.11	2.11	2.11	2.15
	0.9	2.44	2.29	2.23	2.16	2.13	2.14	2.20	3.68	3.52	3.44	3.30	3.22	3.23	3.35
	-0.9	0.54	0.54	0.56	0.58	0.61	0.62	0.63	0.52	0.53	0.53	0·55	0.57	0.57	0.58
	-0.7	0.71	0.70	0.70	0.71	0.72	0.73	0.73	0.65	0.65	0.65	0.65	0.66	0.66	0·67
	-0.5	0.80	0.80	0.79	0.80	0.80	0.81	0.81	0.74	0.73	0.73	0.73	0.74	0.74	0.75
0.25	0.0	1.03	1.01	1.00	1.00	1.00	1.01	1.02	1.01	1.00	1.00	1.00	1.00	1.01	1.02
	.05	1.41	1.36	1.34	1.33	1.33	1.64	1,36	1.62	1.58	1.57	1.56	1 56	1.57	1.61
	0.7	1.72	1.64	1.61	1.57	1.57	1.58	1.62	2.17	2.10	2.07	2.03	2.01	2.03	2.11
	0.9	2.51	2.25	2.15	2.03	1.98	2.00	2.11	3.60	3.34	3.22	3.01	2.87	2.88	3.09

	−0.9	0.58	0.61	0.63	0.68	0.72	0.73	0.76	0.56	0.58	0.60	0.62	0.64	0.66	0.68
	−0.7	0.74	0.74	0.74	0.76	0.78	0.80	0.82	0.69	0.68	0.69	0.70	0.71	0.72	0.75
	−0.5	0.85	0.82	0.82	0.82	0.84	0.86	0.88	0.78	0.76	0.76	0.77	0.78	0.79	0.82
1.0	0.0	1.08	1.03	1.01	1.00	1.01	1.03	1.07	1.05	1.02	1.01	1.00	1.01	1.03	1.09
	0.5	1.47	1.34	1.30	1.26	1.27	1.30	1.37	1.58	1.49	1.47	1.43	1.44	1.47	1.60
	0.7	1.76	1.56	1.50	1.43	1.42	1.46	1.57	2.02	1.87	1.82	1.74	1.72	1.76	1.96
	0.9	2.33	1.94	1.12	1.68	1.61	1.63	1.81	2.88	2.55	2.44	2.26	2.09	2.08	2.35
	−0.9	0.68	0.73	3.76	0.80	0.84	0.87	0.94	0.67	0.70	0.73	0.74	0.77	0.80	0.89
	−0.7	0.84	0.82	0.82	0.85	0.88	0.91	0.98	0.79	0.78	0.78	0.79	0.82	0.85	0.95
	−0.5	0.94	0.89	0.88	0.89	0.92	0.95	1.03	0.88	0.84	0.84	0.84	0.87	0.90	1.01
4.	0.0	1.17	1.05	1.02	1.00	1.02	1.06	1.17	1.10	1.03	1.01	1.00	1.02	1.01	1.23
	0.5	1.46	1.25	1.19	1.14	1.15	1.19	1.36	1.43	1.30	1.26	1.23	1.23	1.28	1.52
	0.7	1.63	1.36	1.28	1.22	1.21	1.24	1.42	1.63	1.45	1.41	1.36	1.34	1.37	1.63
	0.9	1.84	1.49	1.39	1.31	1.26	1.26	1.39	1.89	1.65	1.59	1.52	1.44	1.42	1.56
	−0.9	0.79	0.83	0.85	0.88	0.92	0.96	1.09	0.80	0.81	0.82	0.84	0.87	0.92	1.09
	−0.7	0.91	0.89	0.89	0.91	0.94	0.99	1.12	0.87	0.86	0.86	0.87	0.90	0.95	1.13
	−0.5	1.00	0.93	0.93	0.93	0.96	1.01	1.15	0.54	0.90	0.90	0.90	0.93	0.98	1.18
10.0	0.0	1.19	1.05	1.02	1.00	1.02	1.06	1.23	1.12	1.02	1.01	1.00	1.02	1.07	1.30
	0.5	1.39	1.17	1.11	1.08	1.08	1.11	1.29	1.31	1.17	1.14	1.12	1.12	1.16	1.39
	0.7	1.47	1.22	1.16	1.11	1.10	1.12	1.28	1.40	1.24	1.20	1.18	1.16	1.18	1.38
	0.9	1.57	1.20	1.28	1.15	1.13	1.13	1.19	1.50	1.32	1.28	1.24	1.20	1.19	1.26

TABLE 4.1

Relative efficiency of $\overline{D}^{(2)}$ w.r.t. $\overline{d}^{(2)}$ for different values of P_b, P_w ϕ and q for m=4.

ϕ	P_b	q+0.5							q=0.75						
		P_w=—0.9	—0.7	—0.5	0.0	0.5	0.7	0.9	P_w——0.9	—0.7	—0.5	0.0	0.5	0.7	0.9
	—0.9	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.62	0.62	0.62	0.62	0.62	0.62	0.62
	—0.7	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	—0.5	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.74	0.74	0.74	0·74	0.74	0.74	0.74
0.1	0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.01
	0.5	1.35	1.35	1,34	1.34	1.34	1.34	1.35	1.60	1.60	1.60	1.59	1.59	1.60	1.61
	0.7	1.60	1.59	1.59	1.57	1.57	1.57	1.59	2.14	2.13	2.12	2.10	2.09	2.10	2.13
	0.9	2.15	2.12	1.09	2.0[illegible]	2.01	2.02	2.08	3.42	3.37	3.32	3.19	3.10	3.12	3.23
	—0.9	0.72	0.72	0.72	0.72	0.72	0.72	0.73	0.63	0.63	0.63	0.63	0.63	0.63	0.63
	—0.7	0.76	0.76	0.76	0.76	0.76	0.77	0.77	0.68	0.68	0.68	0.68	0.68	0.63	0.69
	—0.5	0.[illegible]2	0.81	0.81	0.81	0.82	0.82	0.83	0.74	0.74	9.74	0.74	0.75	0.75	0.76
9.25	0.0	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.00	1.00	1.00	1.00	1.00	1.01	1.03
	0.5	1.35	1.34	1.33	1.32	1.32	1.33	1.35	1.58	1.58	1.57	1.57	1.56	1.57	1.61
	0.7	1.60	1.58	1.57	1.54	1.53	1.54	1.58	2.09	2.07	2.06	2.02	2.00	2.02	2.10
	0.9	2.11	2.06	2.02	1.91	2.86	1.89	2.00	3.27	3.20	3.13	2 93	2.77	2.78	3.00

	—0.7	0.75	0.59	0.75	0.75	0.76	0.76	0.78	0.67	0.66	0.66	0.66	0.66	0.67	0.70
	—0.7	0.79	0.79	0.79	0.79	0.80	0.81	0.83	0.71	0.71	0.71	0.71	0.72	0.73	0.75
	—0.5	0.84	0.84	0.84	0.84	0.85	0.86	0.88	0.77	0.77	0.77	0.77	0.78	0.79	0.83
1.0	0.0	1.01	1.01	1.01	1.90	1.01	1.03	1.07	1.01	1.00	1.00	1.00	1.01	1.03	1.09
	0 5	1.31	1.30	1.29	1.26	1 26	1.29	1.36	1.48	1.47	1.46	1.44	1.44	1.48	1.61
	0.7	1.51	1.49	1.47	1.41	1.39	1.43	1.53	1.84	1.83	1.81	1.74	1.71	1.75	1.95
	0.9	1.84	1.80	1.75	1.63	1.54	1.56	1.74	2.49	2.46	2.41	2.24	2.04	2.02	2.31
	—0.9	0.8	0.82	0.82	0.82	0.85	0.88	0.95	0.76	0.76	0.75	0.75	0.77	0.80	0.89
	—0.7	0.86	0.86	0.85	0.85	0.88	0.92	0.99	0.80	0.80	0.80	0.79	0.82	0.85	0.95
	—0.5	0.90	0.89	0.89	0.89	0.92	0.96	0.04	0.35	0.85	0.84	0.84	0.87	0.91	1.02
4	0.0	1.02	1.01	1.01	1.00	1.03	1.07	1.18	1.00	1.01	1.00	1.00	1.02	1.07	1.24
	0.5	1.19	1.18	1.17	1.14	1.15	1.19	1.34	1.25	1.25	1.25	1.23	1.23	1.29	1.54
	0.7	1.28	1.27	1.27	1.22	1.20	1.23	1.39	1.39	1.40	1.39	1.36	1.33	1.37	1.63
	0.9	1.39	1.38	1.36	1.30	1.24	1.23	1.36	1.59	1.58	1.57	1.52	1.43	1.40	1.55
	—0.9	0.89	0.89	0.89	0.89	0.92	0.98	1.10	0.84	0.84	0.84	0.84	0.87	0.92	1.10
	—0.7	0.92	0.91	0 91	0.91	0.94	1.00	1.13	0.88	0.87	0.87	0.87	0.90	0.95	1.15
	—0.5	0.94	0.94	0.94	0.93	0.97	1.02	1.16	0.91	0.91	0.91	0.90	0.93	0.99	1.20
10	0.0	1.01	1.01	1.01	1.00	1.02	1 08	1.23	1.00	1.00	1.00	1.00	1.02	1.08	1.32
	0.5	1.10	1.10	1.09	1.08	1.08	1.11	1.27	1.12	1.13	1.13	1.12	1.12	1.16	1.40
	0.7	1.14	1.14	1.13	1.11	1.10	1.12	1.25	1.18	1.19	1.19	1.18	1.16	1.18	1.38
	0 9	1.18	1.18	1.18	1.15	1.12	1.11	1.18	1.25	1.26	1.26	1.24	1.20	1.18	1.25

TABLE 4.1

Relative efficiency of $\overline{Z}^{(2)}$ *w.r.t.* $\overline{z}\,(2)$ *for different values of* P_b, P_w, ϕ *and* q *for* $m=4$

ϕ	P_b	$q=.50$							$q=.75$						
		$P_w=-0.9$	-0.7	-0.5	0.0	0.5	0.7	0.9	$P_w=-0.9$	-0.7	-0.5	0.0	0.5	0.7	0.9
0.1	−0.9	0.81	0.81	0.82	0.82	0.83	0.83	0.84	0.79	0.80	0.80	0.81	0.82	0.82	0.83
	−0.7	0.90	0.90	0.90	0.90	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.92	0.92	0.92
	−0.5	0.95	0.95	0.95	0.95	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.97	0.97
	0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.5	0.96	0.96	0.96	0.95	0.95	0.95	0.95	0.97	0.97	0.96	0.96	0.96	0.96	0.96
	0.7	0.91	0.91	0·91	0.90	0.90	0.90	0.90	0.92	0.95	0.92	0.91	0.91	0.91	0.91
	0.9	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.83	0.82	0.82	0.81	0.80	0.80	0.79
0.25	−0.9	0.82	0.82	0.82	0.84	0.85	0.86	0.87	0.80	0.80	0.81	0.83	0.85	0.86	0.87
	−0.7	0.90	0.90	0.90	0.91	0.92	0.93	0.93	0.91	0.91	0.91	0.92	0.93	0.94	0.95
	−0.5	0.95	0.95	0.95	0.96	0.96	0.97	0.98	0.97	0.96	0.96	0.96	0.97	0.98	0.98
	0.0	1.01	1.01	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.01
	0.5	0.98	0.97	0.96	0.96	0.95	0.95	0.96	0.98	0.98	0.97	0.96	0.96	0.96	0.97
	0.7	0.93	0.93	0.92	0.91	0.90	0.90	0.90	0.95	0.94	0.93	0.92	0.91	0.91	0.91
	0.9	0.87	0.86	0.85	0.85	0.82	0.82	0.82	0.87	0.86	0.85	0.83	0.81	0.80	0.80

	−0.9	0.84	0.85	0.86	0.89	0.93	0.95	0.98	0.82	0.84	0.85	0.90	0.94	0.96	0.99
	−0.7	0.92	0.91	0.92	0.94	0.97	0.99	1.01	0.93	0.92	0.93	0.95	0.97	0.99	1.02
	−0.5	0.97	0.96	0.96	0.97	0.99	1.01	1.03	0.98	0.97	0.97	0.98	0.99	1.01	1.04
1.0	0.0	1.03	1.02	1.01	1.00	1.01	1.02	1.03	1.04	1.02	1.01	1.00	1.01	1.02	1.04
	0.5	1.03	1.01	0.99	0.97	0.96	0.96	0.97	1.04	1.01	0.99	0.98	0.97	0.97	0.98
	0.7	1.01	0.99	0.97	0.94	0.92	0.92	0.92	1.02	0.99	0.97	0.95	0.93	0.92	0.93
	0.9	0.98	0.95	0.93	0.89	0.86	0.85	0.84	0.99	0.96	0.94	0.90	0.85	0.84	0.82
	−0.9	0.89	0.90	0.92	0.96	1.02	1.05	1.10	0.88	0.90	0.92	0.97	1.01	1.05	1.12
	−0.7	0.95	0.94	0.95	0.98	1.02	1.05	1.10	0.96	0.95	0.96	0.98	1.02	1.05	1.11
	−0.5	0.99	0.98	0.97	0.99	1.03	1.05	1.10	1.01	0.98	0.98	0.99	1.02	1.05	1.11
4	0.0	1.06	1.03	1.01	1.00	1.01	1.03	1.06	1.08	1.03	1.01	1.00	1.01	1.03	1.08
	0.5	1.10	1.05	1.03	0.99	0.97	0.98	0.99	1.11	1.05	1.02	0.99	0.98	0.98	1.01
	0.7	1.10	1.05	1.02	0.98	0.95	0.94	0.95	1.11	1.05	1.02	0.98	0.96	0.95	0.96
	0.9	1.10	1.05	1.02	0.96	0.92	0.90	0.89	1.12	1.05	1.01	0.97	0.92	0.90	0.88
	−0.9	0.94	0.94	0.96	0.99	1.04	0.07	1.12	0.93	0.94	0.96	0.99	1.03	1.06	1.15
	−0.7	0.97	0.97	0.97	0.99	1.03	1.06	1.11	0.98	0.97	0.98	0.99	1.03	1.06	1.14
	−0.5	1.00	0.99	0.98	1.00	1.03	1.05	1.10	1.02	0.99	0.99	1.00	1.02	1.05	1.13
10	0.0	1.06	1.03	1.01	1.00	1.01	1.03	1.06	1.09	1.03	1.01	1.00	1.01	1.03	1.09
	0.5	1.10	1.05	1.03	1.00	0.98	1.99	1.00	1.13	1.05	1.02	1.00	0.99	0.99	1.02
	0.7	1.11	1.06	1.03	0.99	0.97	0.97	0.97	1.14	1.06	1.03	0.99	0.98	0.97	0.98
	0.9	0.12	1.07	1.04	0.99	0.96	0.94	0.94	1.15	1.06	1.03	0.99	0.96	0.94	0.93